Muon decay: From Michel over SMEFT and LEFT to BSM

Exposé for a Master's thesis

Jakob Moritz

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Since the experimental discovery of the Higgs boson in 2013, not much has changed for particle physics on the base level: Neither has the zoo of elementary particles welcomed new inhabitants nor have the long-time residents started to behave differently. The Standard Model (SM) of physics, which directs the microscopic ballet of nature's ensemble of fundamental building blocks by Lorentz-invariant quantum field operators, seems to be complete. Rivaling models, hoping to exceed the SM in terms of mathematical beauty, simplicity, naturalness, or gravitas, hitherto have failed to make correct predictions, any predictions at all, or at least such ones that could be tested by reasonable experimental means anytime soon.

Is the search for physics beyond the SM over and plans for new particle colliders, without anything specific to look for, blind groping around in the fog? The chances are high that this is a bit too pessimistic a view. One of the biggest virtues of the SM has not yet been fully actualized: The virtue that lies in its own becoming. As a matter of fact, some of the particle interactions it currently models as mediated by virtual heavy bosons, for example in beta decay, were successfully described by effective field theories long before. The whole process of beta decay, where a neutron decays into a proton, an electron, and an antineutrino, was already remarkably well characterized as a direct interaction between four fermions [1]. Only the need to restore unitarity at higher energies (as well as gauge theoretic considerations) made the refinement of the 4-Fermi action by a mediating W^- boson necessary.

If a low energy effective field theory, however, guided physicists to the present day SM, could this model not be itself an effective theory, merely enveloping a plethora of yet unknown particles? The idea of the Standard Model effective field theory (SMEFT) has met with growing interest in the last decade, as it provides a transparent and unbiased guidance on the quest for new physics. Instead of envisioning completely new models from scratch, the SMEFT is oriented towards a simple principle: New physics will show up first in the form of effective SM operators. In its present-day canonical form, however, the SM consists of the most general renormalizable quantum field theory that could be constructed from the SM field content respecting Lorentz invariance and $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance. So the current SM Lagrangian contains all possible symmetry conserving combinations of the stipulated set of matter and force mediating fields - but only up to mass dimension 4. All higher-dimensional operators were deemed inadmissible, as they would need infinitely many counterterms to render the theory finite with respect to ultraviolet divergences.

However, if the Standard Model is viewed as an effective field theory that is only valid up to an energy scale Λ, the need for a complete and fully renormalizable theory is eliminated: Physic's potentiality to keep changing its dynamical content at higher energies is simply acknowledged. Higher-dimensional operators therefore enter the Lagrangian of the SMEFT, which encode new physics and whose effects are constrained by experimental data on lower energies.

In this Master thesis project, we want to evaluate the effects of SMEFT dimension 6 operators on the Michel parameters of the muon decay, which determine the form of the emitted electron's energy spectrum. They were first introduced [4] in the context of 4-Fermi theory to trace the effects of different (scalar, vector, tensor) formulations of 4-Fermi interaction to the electron spectrum and take on specific values in the Standard Model's V-A formulation. Present-day experiments strongly support the V-A weak interaction of the Standard Model [5], but this should only encourage further investigation: Which dimension 6 SMEFT operators would alter the Michel parameters and therefore to what extent are their effects constrained by experimental data? In order to observe the SMEFT operators' traces in the effective 4-Fermi interaction, the SMEFT is matched to the LEFT: The low energy effective field theory (LEFT) integrates out the heavy bosons as well as the top quark and reduces to a theory analogous, but more general, than the original fermi theory. Full Matching of the SMEFT onto the LEFT, i.e. adjusting their coefficients so they effectively describe the same amplitudes, has been presented in a paper from 2018 [2]. We use the results to determine Michel parameters for the LEFT and find constraints for the SMEFT. Additionally, we use the dictionary between SMEFT and BSM fields to find phenomenological limits for novel Beyond Standard Model fields.

Contents

1 Muon decay in LEFT

Seven operators contribute to the muon decay in LEFT [2] for all possible combinations of neutrino output:

$$\mathcal{O}_{\nu e}^{V,LL} = (\bar{\nu}_{Lp}\gamma^{\mu}\nu_{Lr})(\bar{e}_{Ls}\gamma_{\mu}e_{Lt}) \quad \text{in 14 flavor representations} \tag{1}$$

$$\mathcal{O}_{\nu e}^{V,LR} = (\bar{\nu}_{Lp}\gamma^{\mu}\nu_{Lr})(\bar{e}_{Rs}\gamma_{\mu}e_{Rt}) \quad \text{in 14 flavor representations} \tag{2}$$

$$\mathcal{O}_{\nu e}^{S,LL} = (\nu_{Lp}^{T}C\nu_{Lr})(\bar{e}_{Rs}e_{Lt}) \quad \text{in 36 flavor representations} \tag{3}$$

$$\mathcal{O}_{\nu e}^{S,LR} = (\nu_{Lp}^{T}C\nu_{Lr})(\bar{e}_{Ls}e_{Rt}) \quad \text{in 36 flavor representations} \tag{4}$$

$$\mathcal{O}_{\nu e}^{T,LL} = (\nu_{Lp}^{T}C\sigma^{\mu\nu}\nu_{Lr})(\bar{e}_{Rs}\sigma_{\mu\nu}e_{Lt}) \quad \text{in 24 flavor representations} \tag{5}$$

$$\mathcal{O}_{\nu\gamma} = (\nu_{Lp}^{T}C\sigma^{\mu\nu}\nu_{Lr})F_{\mu\nu} \quad \text{in 12 flavor representations} \tag{6}$$

$$\mathcal{O}_{e\gamma} = \bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr}F_{\mu\nu} \quad \text{in 4 flavor representations} \tag{7}$$

2 Matching LEFT to SMEFT coefficients

The matching to the SMEFT [2, p.35] gives:

$$\underset{prst}{\mathcal{O}_{\nu e}^{V,LL}} \implies \underset{prst}{C_{ll}} + \underset{stpr}{C_{ll}} - \frac{\bar{g}_2^2}{2M_W^2}[W_l]_{pt}[W_l]_{rs}^{*} - \frac{\bar{g}_Z^2}{M_W^2}[Z_{\nu}]_{pr}[Z_{e_L}]_{st}^{*} \tag{8}$$

$$\underset{prst}{\mathcal{O}_{\nu e}^{V,LR}} \implies \underset{prst}{C_{ll}} - \frac{\bar{g}_Z^2}{M_W^2}[Z_{\nu}]_{pr}[Z_{e_L}]_{st}^{*} \tag{9}$$

$$\underset{prst}{\mathcal{O}_{\nu e}^{S,LL}} \implies 0 \tag{10}$$

$$\underset{prst}{\mathcal{O}_{\nu e}^{S,LR}} \implies 0 \tag{11}$$

$$\underset{prst}{\mathcal{O}_{\nu e}^{T,LL}} \implies 0 \tag{12}$$

$$\underset{pr}{\mathcal{O}_{\nu\gamma}} \implies 0 \tag{13}$$

$$\underset{pr}{\mathcal{O}_{e\gamma}} \implies \frac{1}{\sqrt{2}}\left(-\underset{pr}{C_{eW}}\bar{s} + \underset{pr}{C_{eB}}\bar{c}\right)v_T \tag{14}$$

We insert the appropriate expressions and drop all higher powers - and we do this for all flavor representations. Let's begin with the operator $\mathcal{O}^{V,LL}_{\underset{prst}{\nu e}}$:

$$
\begin{aligned}
L^{V,LL}_{\underset{1112}{\nu e}} &= C_{\underset{1112}{ll}} + C_{\underset{1211}{ll}} - 2C^{(3)}_{\underset{12}{Hl}} && (15)\\
L^{V,LL}_{\underset{1212}{\nu e}} &= C_{\underset{1212}{ll}} + C_{\underset{1212}{ll}} && (16)\\
L^{V,LL}_{\underset{1312}{\nu e}} &= C_{\underset{1312}{ll}} + C_{\underset{1213}{ll}} && (17)\\
cL^{V,LL}_{\underset{1312}{\nu e}} &= C_{\underset{1312}{ll}} + C_{\underset{1213}{ll}} && (18)\\
L^{V,LL}_{\underset{2112}{\nu e}} &= -\frac{2}{v_T^2} + C_{\underset{2112}{ll}} + C_{\underset{1221}{ll}} - 2C^{(3)}_{\underset{22}{Hl}} - 2C^{(3)}_{\underset{11}{Hl}} && (19)\\
&\ldots && (20)
\end{aligned}
$$

3 Matching SMEFT coefficients to BSM models

From [3] we use the tree-level BSM dictionary. The SMEFT dim 6 Wilson coefficient $C_{\underset{2112}{ll}}$ for example is produced by four simplified BSM models [3, p.17, 19]: S_2, S_6, V_1, V_4. The Lagrangian for S_2, for example, reads as:

$$
\Delta\mathcal{L}^{S_2} = \mathcal{C}^{rp}_{HS_2^\dagger S_4}\epsilon^{ji}H_j(S_{4r})S^\dagger_{2p} - \mathcal{C}^{spr}_{S_2^\dagger S_5 S_6}(S_{5r})^I(S_{6s})^I S^\dagger_{2p} - \mathcal{D}^{rsp}_{LLS_2}\epsilon^{ij}[(l_r)_i C(l_s)_j)]S_{2p} \tag{21}
$$

The Matching of the Wilson coefficients reads as:

$$
C_{\underset{f_1 f_2 f_3 f_4}{ll}} = -\frac{\mathcal{D}^{f_1 f_2 p_1}_{LLS_2}\mathcal{D}^{f_4 f_3 p_1 *}_{LLS_2}}{2M^2_{S_2}} + \frac{\mathcal{D}^{f_2 f_1 p_1}_{LLS_2}\mathcal{D}^{f_4 f_3 p_1 *}_{LLS_2}}{2M^2_{S_2}} \tag{22}
$$

4 From Michel parameters to BSM constraints

The differential rate for muon decay looks like:

$$
\begin{aligned}
\mathrm{d}^2\Gamma_{\mu^-\to e^- + \overline{\nu}_e + \nu_\mu} =& \frac{G_F^2 m_1^5}{192\pi^3}\left[6(1-x) + 4\rho\left(\frac{4x}{3} - 1\right)\right.\\
&\left. -2\xi\cos\vartheta\left(1 - x - 2\delta\left(\frac{4x}{3} - 1\right)\right)\right] x^2\mathrm{d}x\sin\theta\mathrm{d}\vartheta
\end{aligned} \tag{23}
$$

For the V-A-interaction of the Standard Model we have $\rho = \delta = \frac{3}{4}$ and $\xi = 1$, which gives us the well known:

$$
\frac{\mathrm{d}^2\Gamma}{\mathrm{d}x\mathrm{d}\cos\vartheta} = \frac{G_F^2 m_1^5}{192\pi^3}[3 - 2x - \cos\vartheta(2x-1)]x^2 \tag{24}
$$

In this project, we calculate the Michel parameters in the LEFT operator basis and restrict (via SMEFT) the novel BSM models.

References

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